

UNIVERSITI MALAYA
UNIVERSITY OF MALAYA

PEPERIKSAAN IJAZAH SARJANA MUDA KEJURUTERAAN
EXAMINATION FOR THE DEGREE OF BACHELOR OF ENGINEERING

SESI AKADEMIK 2018/2019 : SEMESTER I
ACADEMIC SESSION 2017/2018 : SEMESTER I

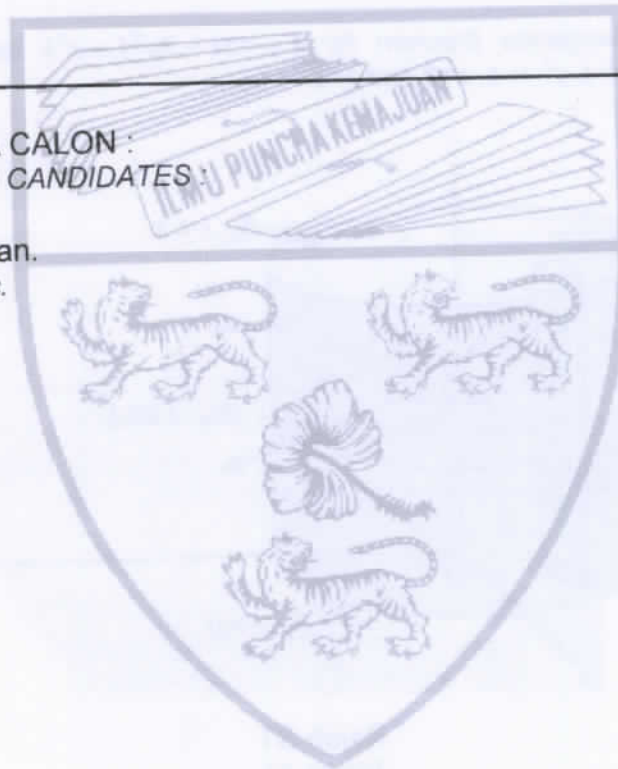
KIX1002 : Matematik Kejuruteraan 2
Engineering Mathematics 2

Januari 2019
January 2019

Masa : 2 jam
Time : 2 hours

ARAHAN KEPADA CALON :
INSTRUCTIONS TO CANDIDATES :

Jawab semua soalan.
Answer all questions.



(Kertas soalan ini mengandungi 4 soalan dalam 7 halaman yang dicetak)
(This question paper consists of 4 questions on 7 printed pages)

Soalan 1
Question 1

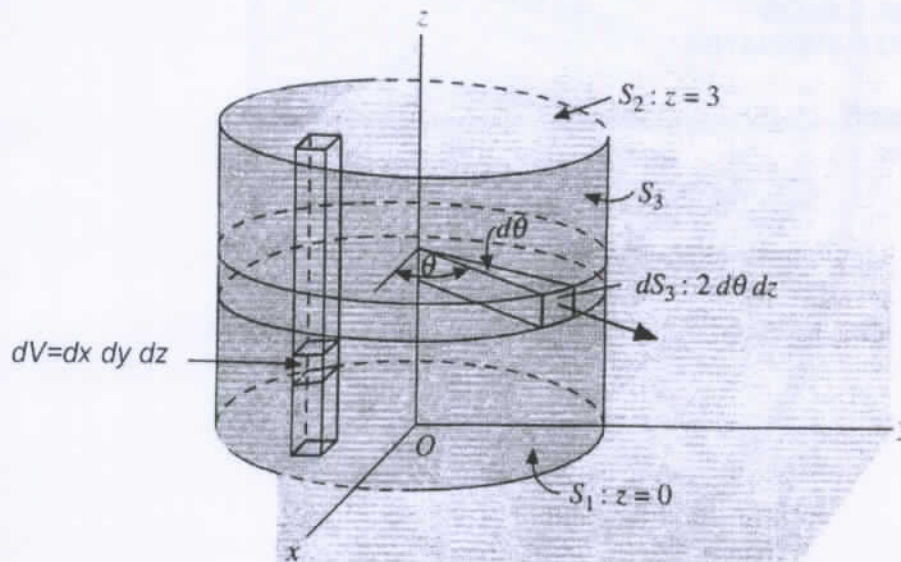
- (a) Cari medan vektor fluks $F(x, y, z) = -y\mathbf{i} + x\mathbf{j} - z\mathbf{k}$ yang melalui sfera unit $x^2 + y^2 + z^2 = 1$ yang mempunyai orientasi ke bawah.

Find the flux of vector field $F(x, y, z) = -y\mathbf{i} + x\mathbf{j} - z\mathbf{k}$ through the unit sphere $x^2 + y^2 + z^2 = 1$ that has downward orientation.

(7 markah / marks)

- (b) Sahkan teorem penyelewengan untuk $A = 4x\mathbf{i} - 2y^2\mathbf{j} + z^2\mathbf{k}$ diambil alih rantau yang dibatasi oleh $x^2 + y^2 = 4$, $z=0$ and $z=3$.

Verify the divergence theorem for $A = 4x\mathbf{i} - 2y^2\mathbf{j} + z^2\mathbf{k}$ taken over the region bounded by $x^2 + y^2 = 4$, $z=0$ and $z=3$.



Rajah S1
Figure S1

(8 markah / marks)

Soalan 2
Question 2

- (a) Cari isipadu dibawah $z = \sqrt{4 - r^2}$ di atas bulatan suku di dalam $x^2 + y^2 = 4$ di dalam kuadran pertama.

Find the volume under $z = \sqrt{4 - r^2}$ above the quarter circle inside $x^2 + y^2 = 4$ in the first quadrant.

(7 markah / marks)

(b) Cari $\iiint_E 16z \, dV$ di mana E ialah bahagian atas sfera $x^2 + y^2 + z^2 = 1$

Find $\iiint_E 16z \, dV$ where E is the upper half of the sphere $x^2 + y^2 + z^2 = 1$

(8 markah / marks)

Soalan 3

Question 3

(a) Persamaan pembezaan berikut mempunyai satu titik tunggal di $x = 0$, dapatkan persamaan hubungan perulangan.

The following differential equation has a singular point at $x = 0$, find its recurrence relation.

$$x^2 y'' + xy' + x^2 y = 0$$

(8 markah / marks)

(b) Selesaikan persamaan di bawah dengan menggunakan transformasi Laplace.

Solve the following equation using Laplace transform.

$$y(t) - \int_0^t y(\tau) \cosh(t - \tau) d\tau = e^t$$

(7 markah / marks)

Soalan 4

Question 4

Diberikan suatu persamaan gelombang

Given a wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad 0 \leq x \leq 4, \quad t > 0$$

dengan syarat-syarat sempadan

with boundary conditions

$$u(0, t) = 0, \quad u(4, t) = 0, \quad t > 0$$

dan syarat-syarat awal

and initial conditions

$$u(x, 0) = f(x) = \sin \frac{\pi x}{4}, \quad 0 \leq x \leq 4$$

$$\frac{\partial u}{\partial t} \Big|_{t=0} = g(x) = \begin{cases} 0, & 0 \leq x \leq 1 \\ a, & 1 \leq x \leq 3 \\ 0, & 3 \leq x \leq 4 \end{cases}$$

- (a) Dengan menggunakan kaedah pemisahan pembolehubah, dapatkan dua persamaan perbezaan biasa (ODE). Nyatakan andaian yang dibuat dan gunakan $-\lambda$ sebagai pemalar pemisahan.

Using separation of variables method, obtain two ordinary differential equations (ODEs). State the assumption made and use $-\lambda$ as the separation constant.

(3 markah / marks)

- (b) Dengan mempertimbangkan tiga kes: $\lambda = 0$, $\lambda = -\alpha^2 < 0$ dan $\lambda = \alpha^2 > 0$, selesaikan persamaan perbezaan tersebut. Kemudian pertimbangkan satu kes untuk mendapatkan nilai eigen dan fungsi eigen.

By considering three cases: $\lambda = 0$, $\lambda = -\alpha^2 < 0$, and $\lambda = \alpha^2 > 0$, solve the differential equations. Then, consider one case to obtain the eigenvalue and eigenfunction.

(7 markah / marks)

- (c) Selesaikan masalah nilai sempadan dengan mempertimbangkan syarat-syarat awal dan menggunakan siri Fourier.

Solve the boundary value problem by considering the initial conditions and using Fourier series.

(5 markah / marks)

**TAMAT
END**

Lampiran A / Appendix A

Koordinat Cartesian ke Sfera
Cartesian to Spherical coordinates

$$x = r \sin \theta \cos \phi$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

$$dS = r^2 \sin \theta \, d\theta \, d\phi$$

$$dV = r^2 \sin \theta \, d\theta \, d\phi \, dz$$

Koordinat Cartesian ke Sylinder
Cartesian to Cylindrical coordinates

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$z = z$$

$$dV = r \, dr \, d\theta \, dz$$

Jadual Jelmaan Laplace
Laplace Transform Table

$f(t)$	$F(s)$	$f(t)$	$F(s)$
1	$\frac{1}{s}$	$af(t) + bg(t)$	$aF(s) + bG(s)$
$\delta(t)$	1	$u(t - a)$	$\frac{e^{-as}}{s}$
t	$\frac{1}{s^2}$	$\delta(t - a)$	e^{-as}
$t^n, n = 1, 2, 3, \dots$	$\frac{n!}{s^{n+1}}$	$f(t - a)u(t - a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s - a}$	$e^{at}f(t)$	$F(s - a)$
te^{at}	$\frac{1}{(s - a)^2}$	$\frac{df}{dt}$	$sF(s) - f(0)$
$t^n e^{at}, n = 1, 2, 3, \dots$	$\frac{n!}{(s - a)^{n+1}}$	$\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0) - f'(0)$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$	$\frac{d^n f}{dt^n}$	$s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$	$\int_0^t f(\tau) \, d\tau$	$\frac{1}{s}F(s)$

$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$	$tf(t)$	$-\frac{d}{ds} F(s)$
$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2}$	$\frac{f(t)}{t}$	$\int_s^\infty F(s) ds$
$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2}$	$f(t) * g(t)$	$F(s)G(s)$
$\cosh \omega t$	$\frac{s}{s^2 - \omega^2}$		

Lampiran B / Appendix B

The Fourier series of a function f defined on the interval $(-p, p)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{p} x + b_n \sin \frac{n\pi}{p} x \right)$$

where

$$a_0 = \frac{1}{p} \int_{-p}^p f(x) dx$$

$$a_n = \frac{1}{p} \int_{-p}^p f(x) \cos \frac{n\pi}{p} x dx$$

$$b_n = \frac{1}{p} \int_{-p}^p f(x) \sin \frac{n\pi}{p} x dx$$

The Fourier Cosine series of an even function on the interval $(-p, p)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{p} x$$

where

$$a_0 = \frac{2}{p} \int_0^p f(x) dx$$

$$a_n = \frac{2}{p} \int_0^p f(x) \cos \frac{n\pi}{p} x dx$$

The Fourier Sine series of an odd function on the interval $(-p, p)$ is

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{p} x$$

where

$$b_n = \frac{2}{p} \int_0^p f(x) \sin \frac{n\pi}{p} x dx$$

Trigonometric Identities

$$\sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$$

$$\cos x \cos y = \frac{1}{2} [\cos(x + y) + \cos(x - y)]$$

$$\sin x \cos y = \frac{1}{2} [\sin(x + y) + \sin(x - y)]$$

$$\cos^2 x = \frac{1}{2} [1 + \cos 2x]$$

$$\sin^2 x = \frac{1}{2} [1 - \cos 2x]$$

TAMAT
END