UNIVERSITI MALAYA
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PEPERIKSAAN IJAZAH SARJANA MUDA KEJURUTERAAN
EXAMINATION FOR THE DEGREE OF BACHELOR OF ENGINEERING

SESI AKADEMIK 2021/2022 : SEMESTER I
ACADEMIC SESSION 2021/2022 : SEMESTER I

KIX1002 : Matematik Kejuruteraan 2
Engineering Mathematics 2

Feb 2022
Feb 2022

Masa : 2 jam
Time : 2 hours

ARAHAN KEPADA CALON:
INSTRUCTIONS TO CANDIDATES:

Calon dikehendaki menjawab SEMUA soalan.
Candidate must answer ALL the questions.

(Kertas soalan ini mengandungi 4 soalan dalam 5 halaman yang dicetak)
(This question paper consists of 4 questions on 5 printed pages)

Soalan 1
Question 1

(a) Pertimbangkan satu permukaan sfera dengan nilai jejari 9. Carikan semua lokasi pada permukaan ini di mana satah tangen masing-masing adalah selari dengan satah rujukan $2x + 3y + 5z = 1$. Seterusnya, tentukan persamaan-persamaan tangen dan garis normal yang berkaitan.
Consider a spherical surface with a radius of 9. Find out all positions on this surface at which the respective tangent planes are parallel to the reference plane $2x + 3y + 5z = 1$. *Then, determine the relevant tangent equations and normal line.*

(9 markah/*marks*)

(b) Pertimbangkan satu situasi aliran rawak bendalir di mana aliran ini boleh dijelaskan dengan medan halaju $\boldsymbol{v}(x, y, z) = (x^3 + y^2 + z)\boldsymbol{i} + (ze^x)\boldsymbol{j} + (xyz - 9xz)\boldsymbol{k}$. Tentukan penyimpangan dan putaran pada titik $(1,1,2)$. Beri tafsiran bagi nilai-nilai yang ditentukan.
Consider a random fluid flow situation in which the flow is described by the velocity field $\boldsymbol{v}(x, y, z) = (x^3 + y^2 + z)\boldsymbol{i} + (ze^x)\boldsymbol{j} + (xyz - 9xz)\boldsymbol{k}$. *Determine the divergence and curl at the point* $(1,1,2)$. *Give interpretation of the values that are determined.*

(6 markah/*marks*)

Soalan 2
Question 2

(a) Anggaran jisim bumi boleh diperolehi dengan mengambil kira bumi sebagai objek sfera dengan ketumpatan yang berubah. Dengan menganggap bahawa ketumpatan berubah secara linear dari pusat, $\rho = 12000\ \mathrm{kgm^{-3}}$, sehingga ke permukaan, $\rho = 3000\ \mathrm{kgm^{-3}}$, kirakan anggaran jisim bumi ini. Gunakan $6 \times 10^6\ \mathrm{m}$ sebagai jejari bumi.
An estimation of the mass of the earth can be obtained by treating the earth as a spherical object with varying density. Assuming that the density varies linearly from the center, $\rho = 12000\ kgm^{-3}$, *to the surface,* $\rho = 3000\ kgm^{-3}$, *calculate this estimated mass of the earth. Use* $6 \times 10^6\ m$ *as the radius of the earth.*

(7 markah/*marks*)

(b) Sahkan Teorem Stokes untuk $\iint_S \mathrm{curl}\, \boldsymbol{F}.d\boldsymbol{S}$ dimana $\boldsymbol{F} = [-y, x, -xyz]$ dan S ialah permukaan kon dengan persamaan $z^2 = y^2 + x^2$ dalam sempadan $0 \leq z \leq 24$. Diberi garis sempadan bergerak secara lawan jam.
Nota: Persamaan parameter permukaan ini ialah $\boldsymbol{r}(u, v) = [v\cos u\,, v\sin u\,, v]$.
Verify Stokes' Theorem for $\iint_S curl\, \boldsymbol{F}.d\boldsymbol{S}$ *where* $\boldsymbol{F} = [-y, x, -xyz]$ *and* S *is a surface of a cone with equation of* $z^2 = y^2 + x^2$ *within the boundary of* $0 \leq z \leq 24$. *It is given that the boundary line moves counter-clockwise.*
Note: Parameterized equation of this surface is $\boldsymbol{r}(u, v) = [v\, cos\, u\,, v\, sin\, u\,, v]$.

(8 markah/*marks*)

Soalan 3
Question 3

Sistem jisim-pegas tanpa redaman mempunyai persamaan pembezaan biasa (ODE) seperti berikut:
An undamped mass-spring system has the following ordinary differential equation (ODE):

$$\frac{d^2x}{dt^2} + 50x = f(t) \tag{3.1}$$

di mana x dan t mewakili sesaran dan masa masing-masing dan $f(t)$ adalah fungsi memaksa.
where x and t represent displacement and time respectively and $f(t)$ is the forcing function.

(a) Fungsi memaksa $f(t)$ tersebut adalah satu fungsi berkala seperti ditunjukkan dalam persamaan (3.2)-(3.3).
The forcing function $f(t)$ is a periodic function as shown in equations (3.2)-(3.3).

$$f(t) = \begin{cases} t, & -1 < t < 0 \\ 0, & 0 < t < 1 \end{cases} \tag{3.2}$$

&

$$f(t) = f(t + 2n)\,,\ n = 1,2,3,\ldots,\infty \tag{3.3}$$

Cari siri Fourier $f(t)$.
Find the Fourier series of $f(t)$.

(8 markah/*marks*)

(b) Penyelesaian khas untuk ODE dari persamaan (3.1) disebabkan oleh fungsi memaksa $f(t)$ dalam bahagian (a) ditunjukkan dalam persamaan (3.4). Tentukan pekali A_0, A_n dan B_n.
The particular solution of ODE from equation (3.1) due to the forcing function $f(t)$ in part (a) is shown in equation (3.4). Determine the coefficients A_0, A_n and B_n.

$$x_p = A_0 + \sum_{n=1}^{\infty}(A_n \cos n\pi t) + \sum_{n=1}^{\infty}(B_n \sin n\pi t)\,,\ n = 1,2,3,\ldots \tag{3.4}$$

(6 markah/*marks*)

(c) Hampirkan x_p dengan menggunakan 2 istilah penghasiltambahan separa, S_2 dalam bahagian (b) pada $t = 0.5$.
Approximate x_p by using 2 terms of partial summation, S_2 in part (b) at $t = 0.5$.

(1 markah/*mark*)

Soalan 4
Question 4

(a) Selesaikan penyelesaian am PDE yang mewakili masalah haba.
Solve the PDE's general solution which represents the heat problem.

$$\frac{\partial u}{\partial t} - 5\frac{\partial^2 u}{\partial x^2} = 0$$

di mana $u(x,t)$ = fungsi suhu
where $u(x,t)$ = temperature function.

(10 markah/*marks*)

(b) Diberikan $\left.\frac{\partial u}{\partial x}\right|_{x=3} = 0$, *u(0,t)=0* untuk *t > 0*, dan *u(x,0)*= 30 untuk *0≤x≤3*. Pertimbangkan syarat sempadan untuk kes 3 ($\lambda = \alpha^2$) sahaja dan sahkan:
Given $\left.\frac{\partial u}{\partial x}\right|_{x=3} = 0$, u(0,t)=0 for t >0, and u(x,0)= 30 for 0≤x≤3. Consider the boundary condition for case 3 ($\lambda = \alpha^2$) only and verify that:

$$u_{total} = \sum_{n=1}^{\infty} e^{-5\left((2n-1)\frac{\pi}{6}\right)^2 t}\left(B_{3,n}sin((2n-1)\frac{\pi}{6}x)\right)$$

Nota: Andaikan kes 1 (λ=0) dan kes 2 ($\lambda = -\alpha^2$) mempunyai penyelesaian khusus sifar di mana $\alpha > 0$. Petunjuk: $cos\left[(2n-1)\frac{\pi}{2}\right] = 0$, di mana $n = 1,2,3,...$
Note: Assume case 1 (λ=0) and case 2 ($\lambda = -\alpha^2$) have zero particular solution where $\alpha > 0$. Hint: $cos\left[(2n-1)\frac{\pi}{2}\right] = 0$, where $n = 1,2,3,...$

(4 markah/*marks*)

(c) Dengan menggunakan syarat awal untuk penyelesaian khusus di bahagian (b), kita memperoleh $\sum_{n=1}^{\infty}\left(B_{3,n}sin((2n-1)\frac{\pi}{6}x)\right) = 30$. Diberikan $B_{3,n} = \frac{2}{L}\int_0^{\tau} f(x)\,sin(Cx)\,dx$, tentukan koefisien L, τ, $f(x)$ *dan* C sahaja tanpa menyelesaikan pengamiran.
By using the initial condition on the particular solution in part (b), we obtain $\sum_{n=1}^{\infty}\left(B_{3,n}sin((2n-1)\frac{\pi}{6}x)\right) = 30$. Given $B_{3,n} = \frac{2}{L}\int_0^{\tau} f(x)\,sin(Cx)\,dx$, determine coefficients L, τ, $f(x)$ and C only without solving the integration.

(1 markah/*mark*)

Lampiran
Appendix

Siri Fourier dan Kembangan Siri Fourier Setengah Julat
Fourier Series and Half-Range Fourier Series Expansion

(i) The Fourier Series of a function $f(x)$ defined on the interval $(-\infty,\infty)$ is given by

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega x + b_n \sin n\omega x)$$

where

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx$$
$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos n\omega x\;dx$$
$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin n\omega x\,dx$$

(ii) The Half-Range Fourier Cosine Series Expansion of a function $f(x)$ defined on the interval $(0,\tau)$ is given by

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega x)$$

where

$$a_0 = \frac{1}{L}\int_{0}^{\tau} f(x)\,dx$$
$$a_n = \frac{2}{L}\int_{0}^{\tau} f(x)\cos n\omega x\;dx$$

(iii) The Half-Range Fourier Sine Series Expansion of a function $f(x)$ defined on the interval $(0,\tau)$ is given by

$$f(x) = \sum_{n=1}^{\infty}(b_n \sin n\omega x)$$

where

$$b_n = \frac{2}{L}\int_{0}^{\tau} f(x)\sin n\omega x\;dx$$

TAMAT
END