KIX1002: Engineering Mathematics 2
Tutorial 7: Frobenius Method

Part I: Using method of reduction of order, find y_2 such that y_1, y_2 form a basis.

1. $2t^2y'' + ty' - 3y = 0,$ $y_1(t) = t^{-1},$ $t \neq 0$
2. $t^2y'' - (t^2 + 2t)y' + (t + 2)y = 0,$ $y_1(t) = t$
3. $y'' + 6y' + 9y = 0,$ $y_1(t) = e^{-3t}$
4. $(x - 1)y'' - xy' + y = 0,$ $y_1(x) = e^x,$ $x > 1$
5. $xy'' - y' + 4x^3y = 0,$ $y_1(x) = \sin x^2,$ $x > 0$

Part II: Discuss whether two Frobenius series solutions exist or do not exist for the following equations.

1. $2x^2y'' + x(x + 1)y' - (\cos x)y = 0$
2. $x^4y'' - (x^2 \sin x)y' + 2(1 - \cos x)y = 0$

Part III: Apply Frobenius Method to find the basis of solutions of the following differential equations.

1. $2xy'' + y' + y = 0$
2. $xy'' + 2y' + xy = 0$
3. $xy'' + (1 - 2x)y' + (x - 1)y = 0$
4. $2ty'' + (1 + t)y' + y = 0$
5. $x(1 - x)y'' - 3xy' - y = 0$